Direct CP, T and/or CPT violations in the $K^0 - \overline{K^0}$ system
- Implications of the recent KTeV results on 2π decays -

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Abstract

The recent results on the CP violating parameters $\mathrm{Re}(\varepsilon'/\varepsilon)$ and $\Delta\phi \equiv \phi_{00} - \phi_{+-}$ reported by the KTeV Collaboration are analyzed with a view to constrain CP, T and CPT violations in a decay process. Combining with some relevant data compiled by the Particle Data Group, we find $\mathrm{Re}(\varepsilon_2 - \varepsilon_0) = (0.85 \pm 3.11) \times 10^{-4}$ and $\mathrm{Im}(\varepsilon_2 - \varepsilon_0) = (3.2 \pm 0.7) \times 10^{-4}$, where $\mathrm{Re}(\varepsilon_I)$ and $\mathrm{Im}(\varepsilon_I)$ represent respectively CP/CPT and CP/T violations in decay of K^0 and $\overline{K^0}$ into a 2π state with isospin I.

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Although it has been well established since 1964 [1] that CP symmetry is violated in the $K^0 - \overline{K^0}$ system, origin or mechanism of CP violation is not well understood yet on the one hand and no evidence of CP violation has been established in any other systems or processes on the other hand. Experimental, phenomenological and theoretical studies of this and related (i.e., T and CPT) symmetries need to be continued with much efforts.

The KTeV Collaboration [2] recently reported

$$\text{Re}(\varepsilon'/\varepsilon) = (2.80 \pm 0.41) \times 10^{-3} , \tag{1a}$$

$$\Delta\phi = (0.09 \pm 0.46)^\circ , \tag{1b}$$

and claimed that the fact $\text{Re}(\varepsilon'/\varepsilon) \neq 0$ definitively established the existence of CP violation in a decay process. In the present note, we like to analyse in detail what the KTeV results imply and to see in particular how well CPT symmetry is tested compared to T symmetry.

The $K^0 - \overline{K^0}$ mixing and 2π decays

Let $|K^0\rangle$ and $|\overline{K^0}\rangle$ be eigenstates of the strong interaction with strangeness $S = +1$ and -1, related to each other by (CP) and (CPT) operations as [3,4]

$$(CP)|K^0\rangle = e^{i\alpha_K}|\overline{K^0}\rangle , \qquad (CPT)|K^0\rangle = e^{i\beta_K}|\overline{K^0}\rangle , \tag{2}$$

where α_K and β_K are arbitrary real parameters. When the weak interaction H_{W} is switched on, K^0 and $\overline{K^0}$ decay into other states, generically denoted as n, and get mixed. The states with definite mass ($m_{S,L}$) and width ($\gamma_{S,L}$; $\gamma_S > \gamma_L$ by definition) are linear combinations of K^0 and $\overline{K^0}$:

$$|K_S\rangle = \frac{1}{\sqrt{|p_S|^2 + |q_S|^2}}(p_S|K^0\rangle + q_S|\overline{K^0}\rangle) , \tag{3a}$$

$$|K_L\rangle = \frac{1}{\sqrt{|p_L|^2 + |q_L|^2}}(p_L|K^0\rangle - q_L|\overline{K^0}\rangle) . \tag{3b}$$

The ratios of the mixing parameters, $q_{S,L}/p_{S,L}$, as well as $\lambda_{S,L} \equiv m_{S,L} - i\gamma_{S,L}/2$, are related to H_{W}; the explicit expressions can be found in the literature [3,5]. We are interested in 2π decays and specifically in the following quantities:

$$\eta_{+-} = |\eta_{+-}|e^{i\phi_{+-}} \equiv \frac{\langle \pi^+\pi^-, \text{outgoing}|H_{\text{W}}|K_L\rangle}{\langle \pi^+\pi^-, \text{outgoing}|H_{\text{W}}|K_S\rangle} , \tag{4a}$$

$$\eta_{00} = |\eta_{00}|e^{i\phi_{00}} \equiv \frac{\langle \pi^0\pi^0, \text{outgoing}|H_{\text{W}}|K_L\rangle}{\langle \pi^0\pi^0, \text{outgoing}|H_{\text{W}}|K_S\rangle} , \tag{4b}$$

$$r \equiv \frac{\gamma_S(\pi^+\pi^-) - 2\gamma_S(\pi^0\pi^0)}{\gamma_S(\pi^+\pi^-) + \gamma_S(\pi^0\pi^0)}, \tag{5}$$

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where $\gamma_{S,L}(n)$ denotes the partial width for $K_{S,L}$ to decay into the final state n.

Parametrization and conditions imposed by CP, T and CPT symmetries

We shall parametrize q_S/p_S and q_L/p_L as [3]

$$\frac{q_S}{p_S} = e^{i\alpha_K} \frac{1 - \varepsilon - \delta}{1 + \varepsilon + \delta} \ , \tag{6a}$$

$$\frac{q_L}{p_L} = e^{i\alpha_K} \frac{1 - \varepsilon + \delta}{1 + \varepsilon - \delta} \ , \tag{6b}$$

and the amplitudes for K^0 and $\overline{K^0}$ to decay into 2π states with isospin $I = 0$ or 2 as [3,6]

$$\langle (2\pi)_I | H_{\mathrm{W}} | K^0 \rangle = F_I (1 + \varepsilon_I) e^{i\alpha_K/2} \ , \tag{7a}$$
$$\langle (2\pi)_I | H_{\mathrm{W}} | \overline{K^0} \rangle = F_I (1 - \varepsilon_I) e^{-i\alpha_K/2} \ . \tag{7b}$$

Our parametrization is very unique in that it is invariant under rephasing of the initial states, $|K^0\rangle$ and $|\overline{K^0}\rangle$. It is however not invariant under rephasing of the final states, $|(2\pi)_I\rangle$. By making use of the phase ambiguity, one may, without loss of generality, set [6]

$$\mathrm{Im}(F_I) = 0 \ . \tag{8}$$

One readily verify [3,6] that CP, T and CPT symmetries impose such conditions as

$$
\begin{array}{llll}
CP \text{ symmetry} & : & \varepsilon = 0, \ \delta = 0, \ \varepsilon_I = 0 \ ; \\
T \text{ symmetry} & : & \varepsilon = 0, \ \mathrm{Im}(\varepsilon_I) = 0 \ ; \\
CPT \text{ symmetry} & : & \delta = 0, \ \mathrm{Re}(\varepsilon_I) = 0 \ .
\end{array}
\tag{9}
$$

Observed and expected smallness of symmetry violation allows one to treat all these parameters as small.

Formulae relevant for analysis

Defining

$$\eta_I = |\eta_I| e^{i\phi_I} \equiv \frac{\langle (2\pi)_I | H_{\mathrm{W}} | K_L \rangle}{\langle (2\pi)_I | H_{\mathrm{W}} | K_S \rangle} \ , \tag{10a}$$

$$\omega \equiv \frac{\langle (2\pi)_2 | H_{\mathrm{W}} | K_S \rangle}{\langle (2\pi)_0 | H_{\mathrm{W}} | K_S \rangle} \ , \tag{10b}$$

one finds [7,8], from Eqs.(3a,b), (6a,b) and (7a,b),

$$\eta_I = \varepsilon - \delta + \varepsilon_I \ , \tag{11a}$$
$$\omega = \mathrm{Re}(F_2)/\mathrm{Re}(F_0) \ , \tag{11b}$$

and, by means of isospin decomposition,

$$\eta_{+-} = \eta_0 + \varepsilon' \; , \tag{12a}$$

$$\eta_{00} = \eta_0 - 2\varepsilon' \; , \tag{12b}$$

$$r = 4\mathrm{Re}(\omega') \; , \tag{13}$$

where

$$\varepsilon' \equiv (\eta_2 - \eta_0)\omega' \; , \tag{14a}$$

$$\omega' \equiv \frac{1}{\sqrt{2}}\omega e^{i(\delta_2 - \delta_0)} \; , \tag{14b}$$

δ_I being the S-wave $\pi\pi$ scattering phase shift for the isospin I state at an energy of the rest mass of K^0. Note that we have treated ω', which is a measure of deviation from the $\Delta I = 1/2$ rule, as well as a small quantity. From Eqs.(12a,b), it follows that

$$\eta_{00}/\eta_{+-} = 1 - 3\varepsilon'/\eta_0 \; , \tag{15}$$

or

$$\mathrm{Re}(\varepsilon'/\eta_0) = (1/3)(1 - |\eta_{00}/\eta_{+-}|) \; , \tag{16a}$$

$$\mathrm{Im}(\varepsilon'/\eta_0) = -(1/3)\Delta\phi \; , \tag{16b}$$

where

$$\Delta\phi \equiv \phi_{00} - \phi_{+-} \; . \tag{17}$$

Implications of the KTeV results

With the help of the formulae derived above, we now look into implications of the latest results reported by the KTeV Collaboration [2]. We first note that, since ε in their notation corresponds exactly to η_0 in our notation,[‡] their results (1a,b) give, either immediately or with the help of Eqs.(16a,b),

$$\mathrm{Re}(\varepsilon'/\eta_0) = (\ \ 2.80 \pm 0.41) \times 10^{-3} \; , \tag{18a}$$

$$\mathrm{Im}(\varepsilon'/\eta_0) = (-0.52 \pm 2.68) \times 10^{-3} \; , \tag{18b}$$

$$|\eta_{00}/\eta_{+-}| = 0.9916 \pm 0.0012 \; . \tag{18c}$$

From Eqs.(11a) and (14a), we immediately conclude that $\varepsilon' \neq 0$ implies that either ε_0 or ε_2 (or both) is $\neq 0$,[§] confirming the assertion that the KTeV result on $\mathrm{Re}(\varepsilon'/\varepsilon)$ established the existence of CP violation in a decay process [2].

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[‡]For the correspondence between our parametrization and the (more conventional) rephasing-dependent parametrizations, see [3,8].

[§]Note that the reverse is however not necessarily true; a nonvanishing but equal value for both ε_0 and ε_2 could yield $\varepsilon' = 0$.

To go one step further, we need to know the value of η_0. Since the KTeV collaboration has not yet reported their results on η_{+-} and η_{00} separately, we shall input the PDG [9] values for η_{+-},

$$|\eta_{+-}| \;=\; (2.285 \pm 0.019) \times 10^{-3} \;, \tag{19a}$$
$$\phi_{+-} \;=\; (43.5 \pm 0.6)^\circ \;, \tag{19b}$$

along with Eqs.(1b) and (18c), into

$$\eta_0 \simeq (2/3)\eta_{+-} + (1/3)\eta_{00}, \tag{20}$$

which follows from Eqs.(12a,b), to get

$$|\eta_0| \;=\; (2.28 \pm 0.02) \times 10^{-3} \;, \tag{21a}$$
$$\phi_0 \;=\; (43.53 \pm 0.94)^\circ \;. \tag{21b}$$

We shall also use the PDG [9] values for $\gamma_S(\pi^+\pi^-)$ and $\gamma_S(\pi^0\pi^0)$ to get, with the help of Eqs.(5) and (13),

$$\mathrm{Re}(\omega') = (1.46 \pm 0.16) \times 10^{-2} \;. \tag{22}$$

In order to interpret Eqs.(18a,b), we derive from Eqs.(14a,b), with the aid of Eqs.(11a,b),

$$\varepsilon'/\eta_0 = -i\mathrm{Re}(\omega')(\varepsilon_2 - \varepsilon_0)e^{-i\Delta\phi'}/[|\eta_0|\cos(\delta_2 - \delta_0)] \;, \tag{23}$$

or

$$\varepsilon_2 - \varepsilon_0 = i(\varepsilon'/\eta_0)|\eta_0|\cos(\delta_2 - \delta_0)e^{i\Delta\phi'}/\mathrm{Re}(\omega') \;, \tag{24}$$

where

$$\Delta\phi' \equiv \phi_0 - \delta_2 + \delta_0 - \pi/2 \;. \tag{25}$$

Inputting Eqs.(18a,b), (21a,b) and (22), and $\delta_2 - \delta_0$ as well, into Eq.(24), we are able to derive constraints to $\mathrm{Re}(\varepsilon_2 - \varepsilon_0)$ and $\mathrm{Im}(\varepsilon_2 - \varepsilon_0)$:

$$\mathrm{Re}(\varepsilon_2 - \varepsilon_0) = (0.85 \pm 3.11) \times 10^{-4} \;, \tag{26a}$$
$$\mathrm{Im}(\varepsilon_2 - \varepsilon_0) = (3.2 \;\pm 0.7\;) \times 10^{-4} \;, \tag{26b}$$

where, as $\delta_2 - \delta_0$, we have tentatively used the Chell-Olsson value, $(-42 \pm 4)^\circ$ [10].

Discussion

Our result (26b) indicates that a combination of the parameters which signal direct CP and T violations, $\mathrm{Im}(\varepsilon_2 - \varepsilon_0)$, is definitely nonzero and of the order of 10^{-4}. The other result (26a) on the other hand indicates that a combination of the parameters which signal direct CP and CPT violations, $\mathrm{Re}(\varepsilon_2 - \varepsilon_0)$, is not well

determined yet; though consistent with being zero, a value comparable to or even larger than $\mathrm{Im}(\varepsilon_2 - \varepsilon_0)$ is not ruled out.

If, instead of the KTeV values, Eqs.(1a,b), one inputs the PDG [9] values also for $|\eta_{00}/\eta_{+-}|$ and $\Delta\phi$,

$$|\eta_{00}/\eta_{+-}| \;=\; 0.9956 \pm 0.0023 \;, \tag{27a}$$

$$\Delta\phi \;=\; (-0.1 \pm 0.8)^\circ \;, \tag{27b}$$

one will get

$$\mathrm{Re}(\varepsilon'/\eta_0) \;=\; (1.5 \pm 0.8) \times 10^{-3} \;, \tag{28a}$$

$$\mathrm{Im}(\varepsilon'/\eta_0) \;=\; (0.6 \pm 4.7) \times 10^{-3} \;, \tag{28b}$$

and

$$\mathrm{Re}(\varepsilon_2 - \varepsilon_0) \;=\; (-0.56 \pm 5.45) \times 10^{-4} \;, \tag{29a}$$

$$\mathrm{Im}(\varepsilon_2 - \varepsilon_0) \;=\; (\;1.8 \;\;\pm 1.0\;\;) \times 10^{-4} \;. \tag{29b}$$

With the help of the Bell-Steinberger relation [11], one may derive constraints to the "indirect" and "mixed" CP, T and/or CPT violating parameters [7,8,12,13]. It turns out that the values of the direct CP/T violating parameter we have obtained, Eqs.(26b) and(29b), are almost one order smaller than those of the indirect and mixed CP/T violating parameters, $\mathrm{Re}(\varepsilon)$ and $\mathrm{Im}(\varepsilon + \varepsilon_0)$, while the constraints on the direct CP/CPT violating parameter we have found, Eqs.(26a) and (29a), are roughly one order weaker than those on the indirect and mixed CP/CPT violating parameters, $\mathrm{Im}(\delta)$ and $\mathrm{Re}(\delta - \varepsilon_0)$.¶

To conclude, we recall that the numerical results (26a,b) and (29a,b) depend much on the value of $\delta_2 - \delta_0$, and that this quantity, which features strong interaction effects, is still not well determined. In order to obtain a better constraint on $\varepsilon_2 - \varepsilon_0$, a better determination of $\delta_2 - \delta_0$, along with a more precise measurement of $\Delta\phi$, are required.

Acknowledgements

¶ε_0 and ε_2 (ε and δ) are referred to as a direct (indirect) parameter here. Note that, as emphasized in [3], classification of symmetry-violating parameters into "direct" and "indirect" ones makes sense only when they are defined in a rephasing-invariant way, i.e., in such a way that they are invariant under rephasing of $|K^0\rangle$ and $|\overline{K^0}\rangle$.

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